Reduced-Order Models of a Large Flexible Spacecraft

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Two reduced-order models of a large flexible spacecraft are proposed in this paper. The method is based on the component mode synthesis. The first one is expressed in terms of the modes of static deformation and the normal modes of vibration of the spacecraft. The reduced-order model can express the dynamic behavior of the spacecraft accurately in a low-frequency region with the proper choice of the modes of static deformation for the spacecraft. The second one is suited to the design of a control system of a spacecraft composed of the main body and subbodies. The reduced-order model is expressed in terms of two sets of the normal modes of vibration, the normal modes of the whole spacecraft and the normal modes of the subbodies. These reduced-order models are illustrated through application to a simple spacecraft model.

I. Introduction

ADVANCED spacecraft are becoming increasingly flexible. Their low natural frequencies fall within the bandwidth of the control system. A dynamic model of this class of spacecraft becomes generally too large for a control designer to cope with. The size of the model must be reduced to perform the control system design. A reduced-order model for this purpose need not express displacements at all the points of the spacecraft in a wide frequency region accurately; it is sufficient that it express displacements at important points to the control system in a certain frequency region. This paper will propose two new reduced-order models of a large flexible spacecraft based on the component mode synthesis.[1,2] Usually, equations of motion for this class of spacecraft are expressed in terms of the normal modes of vibration of the spacecraft, and a reduced-order is derived by deletion of certain elements of the normal modes of vibration. This reduced-order model has some disadvantages. One is as follows: Since a displacement of the spacecraft is expressed in all the normal modes of vibration of the spacecraft, deletion of certain elements of the normal modes of vibration may cause an error in the displacements at important points to the control system, which results in a serious degradation of performance of the control system. In order to overcome this disadvantage, the following reduced-order model will be established: A dynamic model of the spacecraft is expressed in terms of the normal modes of vibration and the modes of static displacement of the spacecraft. A reduced-order model is derived by the deletion of certain components of the normal modes of vibration. By the proper choice of static displacements, this reduced-order model accurately expresses the displacements at certain points of the spacecraft in a low-frequency region. On the other hand, a large flexible spacecraft is generally composed of a main body and subbodies with mission equipment. The control system for this class of spacecraft usually consists of the global and local controllers; the former controls the attitude of the whole spacecraft and the latter the attitude of the subbodies. A reduced-order model for a design of this type of controller must express displacements of the whole spacecraft and the subbodies accurately. A reduced-order model based on the normal modes of vibration of the whole spacecraft is not suited to this purpose. For example, at the design phase, the structures of the subbodies are changed frequently. When the structure of the spacecraft is changed, the reduced-order model requires calculation of the eigenvalue problem with very large dimensions for a wide range of frequency. It requires a lot of time and the cost is high. A reduced-order model that is suited to the spacecraft composed of a main body and subbodies will be established; a dynamic model of the spacecraft is expressed in two sets of the normal modes of vibration, in which one is a set of the normal modes of vibration of the whole spacecraft and the other is a set of the normal modes of vibration of the subbodies. The reduced-order model is derived by the deletion of certain components in each set of the normal modes of vibration. The global controller is designed based on the equations for the normal modes of vibration of the whole spacecraft and the local controller is designed based on the equations for the normal modes of vibration of the subbodies. Moreover, when the structures of the subbodies are changed, this reduced-order model does not require the calculation of the eigenvalue problem with very large dimensions for a wide range of frequency.

II. Spacecraft Model

A spacecraft model is schematically shown in Fig. 1. The reference axes (O–XYZ) are set on the spacecraft. The position of any point in the spacecraft is denoted by a vector $\boldsymbol{r}$. A dynamic model of the spacecraft is derived by the use of a finite-element method (FEM)[3]; divide the spacecraft into small elements and give a number to a nodal point. A displacement of a nodal point i from its nominal position is defined by a column matrix R_i, which has six components corresponding to transverse and rotational displacements. A displacement of a point $\boldsymbol{r}$ is denoted by $u(\boldsymbol{r},t)$, which is expressed in terms of R_i as

$$u(\boldsymbol{r},t) = \sum_{i=1}^{N} R_i^T \Psi_i(\boldsymbol{r}) \tag{1}$$

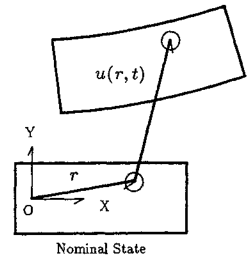

Fig. 1 Generic model of spacecraft.

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where $\Psi_i(r)$ is a column matrix in which the components define the element shape functions related to the displacements R_i. Define column matrices R and $\Psi(r)$ as

$$R^T = [R_1^T, \ldots, R_N^T]$$

$$\Psi^T(r) = [\Psi_1^T(r), \ldots, \Psi_N^T(r)]$$

where $\bullet^T$ denotes a transpose of a matrix, then $u(r,t)$ is expressed as

$$u(r,t) = R^T \Psi(r) \tag{1'}$$

When a displacement $u(r,t)$ is small, a dynamic model of the spacecraft is given by

$$M\ddot{R} + KR = BU$$
$$Y = C_P R + C_v \dot{R} \tag{2}$$

where $M(= M^T > 0)$ is a mass matrix that usually is derived on a lumped mass model, $K(= K^T \geq 0)$ is a stiffness matrix, U is an acutator input, and Y is a sensor output, B is an actuator influence matrix, and C_P and C_V are measurement matrices.

III. Reduced-Order Model

After transformation of the coordinates R to certain coordinates, a reduced-order model is derived. A reduced-order model in terms of the normal modes of vibration of the spacecraft has been widely used.[4-6] This reduced-order model is explained briefly and then new reduced-order models are proposed. Consider the eigenvalue problem

$$KT = MT\Lambda$$
$$T^T MT = I \tag{3}$$

where $\Lambda = \text{diag}(\lambda_i^2)$. The following transformation is introduced:

$$R = TP \tag{4}$$

The substitution of Eq. (4) into Eq. (2) leads to

$$\ddot{P} + \Lambda P = \hat{B}U$$
$$Y = \hat{C}_P P + \hat{C}_V \dot{P} \tag{5}$$

where $\hat{M} = T^T MT$, $\hat{B} = T^T B$, $\hat{C}_P = C_P T$, $\hat{C}_V = C_V T$. On the other hand, the substitution of Eq. (4) into Eq. (1′) leads to

$$u(r,t) = P^T \Phi(r) \tag{6}$$

where $\Phi(r) = T^T \Psi(r)$, in which the ith element is the ith normal mode of vibration of the spacecraft. Equations (5) are expressed in terms of the magnitudes P of the normal modes of vibration. A reduced-order model is derived by the deletion of certain components of the normal modes of vibration.

A. Reduced-Order Model I

The reduced-order model based on the normal mode of vibration of the spacecraft has the following disadvantage. From Eq. (6), a displacement of a point in the spacecraft is found by superimposing the normal modes of vibration. Deletion of certain elements of the normal modes causes an error in the displacement, and this may result in a serious degradation of the control system's performance. In order to overcome this disadvantage, we propose a new reduced-order model. The column matrix R is partitioned as follows:

$$R^T = [R_O^T \; R_I^T] \tag{7}$$

where R_O is chosen to include the displacements that are of prime importance to the control system, i.e., the displacements of the mission equipment and the sensors and actuators, and R_I represents a subset of the displacements that are of little importance to the control system. The column matrices R_O and R_I are referred to as outer and inner coordinates, respectively. The column matrix $\Psi(r)$ is also partitioned as

$$\Psi^T(r) = [\Psi_O^T(r) \; \Psi_I^T(r)] \tag{7'}$$

As a consequence of classifying the column matrix R into two categories, R_O and R_I, Eq. (2) is partitioned as follows:

$$\begin{bmatrix} M_O & 0 \\ 0 & M_I \end{bmatrix} \begin{bmatrix} \ddot{R}_O \\ \ddot{R}_I \end{bmatrix} + \begin{bmatrix} K_O & K_{OI} \\ (\text{sym}) & K_I \end{bmatrix} \begin{bmatrix} R_O \\ R_I \end{bmatrix} = \begin{bmatrix} B \\ 0 \end{bmatrix} U$$
$$Y = [C_p \; 0] \begin{bmatrix} R_O \\ R_I \end{bmatrix} + [C_v \; 0] \begin{bmatrix} \dot{R}_O \\ \dot{R}_I \end{bmatrix} \tag{8}$$

Consider the eigenvalue problem

$$K_I T_I = M_I T_I \Lambda_I$$
$$T_I^T M_I T_I = I \tag{9}$$

where $\Lambda_I = \text{diag}(\lambda_{Ii}^2)$. Define a column matrix R_{IO} as

$$R_{IO} = -K_I^{-1} K_{OI}^T \tag{10}$$

The following transformation is introduced:

$$\begin{bmatrix} R_O \\ R_I \end{bmatrix} = \begin{bmatrix} I & 0 \\ R_{IO} & T_I \end{bmatrix} \begin{bmatrix} R_O \\ P_I \end{bmatrix} \tag{11}$$

The substitution of Eq. (11) into Eq. (1′), together with Eq. (7′), yields

$$u(r,t) = R_O^T \Theta_O(r) + P_I^T \Phi_I(r) \tag{12}$$

where $\Theta_O(r) = \Psi_O(r) + R_{IO}^T \Psi_I(r)$, in which the ith component is the mode of the static deformation by producing a unit displacement on the ith outer coordinate with all other outer coordinates fixed, and $\Phi_I(r) = T_I^T \Psi_I(r)$, in which the ith component is the ith normal mode of vibration with all the outer coordinates fixed. Expression (12) is similar to the so-called Craig–Bampton model.[7] On the other hand, the substitution of Eq. (11) into Eq. (8) yields

$$\begin{bmatrix} \hat{M}_O & \hat{M}_{OI} \\ (\text{sym}) & I \end{bmatrix} \begin{bmatrix} \ddot{R}_O \\ \ddot{P}_I \end{bmatrix} + \begin{bmatrix} \hat{K}_O & 0 \\ 0 & \Lambda_I \end{bmatrix} \begin{bmatrix} R_O \\ P_I \end{bmatrix} = \begin{bmatrix} B \\ 0 \end{bmatrix} U$$
$$Y = [C_P \; 0] \begin{bmatrix} R_O \\ P_I \end{bmatrix} + [C_V \; 0] \begin{bmatrix} \dot{R}_O \\ \dot{P}_I \end{bmatrix} \tag{13}$$

where

$$\hat{M}_O = M_O + R_{IO}^T M_I R_{IO}$$

$$\hat{M}_I = T_I^T M_I T_I$$

$$\hat{M}_{OI} = R_{IO}^T M_I T_I$$

$$\hat{K}_O = K_O + K_{OI} R_{IO} + R_{IO}^T K_{OI}^T + R_{IO}^T K_I R_{IO}$$

Equation (12) is expressed in terms of the outer coordinate R_O, which is the magnitudes of the mode of the static deformation, and the magnitudes of the normal modes of vibration P_I. A reduced-order model is derived by the deletion of certain components of the normal modes P_I that have weak interactions with the outer coordinates. A component R_{Oj} of the outer coordinate R_O affects a component R_{Ii} of the normal

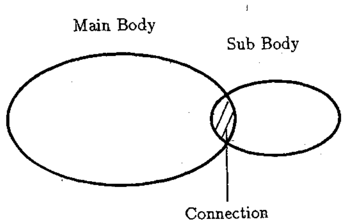

Fig. 2 Spacecraft model composed of main body and subbody.

modes of vibration through the term $\hat{M}_{IOij}\ddot{R}_{Oj}$. In a low-frequency region, a response of the component P_{Ii} may be assumed to be

$$P_{Ii} = -\lambda_{Ii}^{-2} M_{IOij}\ddot{R}_{Oj}$$

The response of the component P_{Ii} affects, in turn, a component $R_{Oj'}$ of the outer coordinate through the term $\hat{M}_{OIj'i}\ddot{P}_{Ii}$. The magnitude J_i of the total interactions of the components of the outer coordinate R_O through the component P_{Ii} of the inner coordinate is given by

$$J_i = \left[\sum_{j',j=1} (\lambda_{Ii}^{-2} \hat{M}_{OIj'i} M_{IOij})^2 \right]^{1/2}$$
$$= [(\Lambda_I^{-1} \hat{M}_{IO} \hat{M}_{OI})(\Lambda_I^{-1} \hat{M}_{IO} \hat{M}_{OI})^T]^{1/2} \qquad (14)$$

From the completeness of the eigenmatrix T_I, the following relation holds

$$\sum_{i=1} J_i^2 = \mathrm{trace}[(R_{IO}^T K_I^{-1} R_{IO})^2]$$

Define the "modal influence parameter" for the ith normal mode as

$$\hat{J}_i = J_i / \{\mathrm{trace}[(R_{IO}^T K_I^{-1} R_{IO})^2]\}^{1/2} \qquad (14')$$

The reduced-order model is derived by the deletion of the normal modes of vibration with small modal influence parameters. The effects of structural damping are introduced to the reduced-order model as a modal damping model after transformation of the reduced-order model to the normal modes of vibration. Since the dynamic interactions of the normal modes P_I with the outer coordinates R_O occur through the inertia forces, the interactions become weak in a low-frequency region. Therefore, in this region, the reduced-order model describes the dynamic behavior of the outer coordinate R_O accurately.

B. Reduced-Order Model II

A large flexible spacecraft is generally composed of a main body and subbodies with mission equipment. The control system of this class of spacecraft consists of the global and local controllers; the former controls an attitude of the whole spacecraft and the latter an attitude of the subbodies with mission equipment. A reduced-order model of this class of spacecraft must, therefore, express the displacements of the whole spacecraft and the subbodies accurately. A new reduced-order model suited to this class of spacecraft is proposed. A generic spacecraft model is shown in Fig. 2; the spacecraft is composed of a main body and subbody. The column matrix R is partitioned as follows:

$$R^T = [R_M^T \, R_C^T \, R_S^T] \qquad (15)$$

where R_M and R_S are the displacements in the main body and subbody, respectively, and R_C represents the displacements in the connection of the main body and subbody. The column matrix $\Psi(r)$ appears in partition form

$$\Psi^T(r) = [\Psi_M^T(r) \, \Psi_C^T(r) \, \Psi_S^T(r)] \qquad (15')$$

A displacement $u(r,t)$ is then expressed as

$$u(r,t) = R_M^T \Psi_M(r) + R_C^T \Psi_C(r) + R_S^T \Psi_S(r) \qquad (15'')$$

As a consequence of classifying the column matrix R in three categories R_M, R_C, R_S, Eq. (2) is partitioned as

$$\begin{bmatrix} M_M & 0 & 0 \\ & M_C & 0 \\ 0 & & M_S \end{bmatrix} \begin{bmatrix} \ddot{R}_M \\ \ddot{R}_C \\ \ddot{R}_S \end{bmatrix} + \begin{bmatrix} K_M & K_{MC} & K_{MS} \\ & K_C & K_{CS} \\ (\mathrm{sym}) & & K_S \end{bmatrix} \begin{bmatrix} R_M \\ R_C \\ R_S \end{bmatrix} = \begin{bmatrix} B_M \\ B_C \\ B_S \end{bmatrix} U \qquad (16)$$

$$Y = [C_{PM} \, C_{PC} \, C_{PS}] \begin{bmatrix} R_M \\ R_C \\ R_S \end{bmatrix} + [C_{VM} \, C_{VC} \, C_{VS}] \begin{bmatrix} \dot{R}_M \\ \dot{R}_C \\ \dot{R}_S \end{bmatrix}$$

Here, we introduce two sets of normal modes of vibration; one is a set of approximate normal modes of vibration for the whole spacecraft and the other a set of normal modes of vibration for the subbody. First, a set of approximate normal modes of vibration for the whole spacecraft is derived. Consider the normal modes of vibration of the main body and the connection

$$\begin{bmatrix} K_M & K_{MC} \\ (\mathrm{sym}) & K_C \end{bmatrix} \begin{bmatrix} T_M \\ T_C \end{bmatrix} = \begin{bmatrix} M_M & 0 \\ 0 & M_C \end{bmatrix} \begin{bmatrix} T_M \\ T_C \end{bmatrix} \Lambda_M$$
$$[T_M^T \, T_C^T] \begin{bmatrix} M_M & 0 \\ 0 & M_C \end{bmatrix} \begin{bmatrix} T_M \\ T_C \end{bmatrix} = I \qquad (17)$$

where $\Lambda_M = \mathrm{diag}(\lambda_{Mi}^2)$. On the other hand, the static deformation R_{SC} of the subbody caused by the displacement T_C of the connection is given by

$$R_{SC} = -K_S^{-1} K_{CS}^T T_C \qquad (18)$$

From Eqs. (17) and (18), the displacement $\Phi_M(r)$ is defined as

$$\Phi_M(r) = T_M^T \Psi_M(r) + T_C^T \Psi_C(r) + R_{SC}^T \Psi_S(r) \qquad (19)$$

The displacement $\Psi_M(r)$ is composed of the normal modes of vibration of the main body and connection and the static deformation of the subbody induced by the normal modes. The displacement $\Phi_M(r)$ becomes an approximation of the normal modes of vibration with low natural frequencies of the whole spacecraft when the mass of the subbody is small compared to that of the whole spacecraft.

Next, a set of normal modes of vibration for the subbody is derived. Consider the eigenvalue problem of the subbody

$$K_S T_S = M_S T_S \Lambda_S$$
$$T_S^T M_S T_S = I \qquad (20)$$

where $\Lambda_S = \mathrm{diag}(\lambda_{Si}^2)$. The displacement $\Phi_S(r)$ is defined as

$$\Phi_S(r) = T_S^T \Psi_S(r) \qquad (21)$$

The displacement $\Phi_S(r)$ is assumed to be an approximation of the normal modes of vibration of the whole spacecraft when the displacement occurs only in the subbody. A displacement $u(r,t)$ is expressed in terms of $\Phi_M(r)$ and $\Phi_S(r)$ as

$$u(r,t) = P_M^T \Phi_M(r) + P_S^T \Phi_S(r) \qquad (22)$$

Expression (22) is similar to the model given by Benfield and Hruda.[2] From Eqs. (15″) and (22), the following transformation is derived:

$$\begin{bmatrix} R_M \\ R_C \\ R_S \end{bmatrix} = \begin{bmatrix} T_M & 0 \\ T_C & 0 \\ R_{SC} & T_S \end{bmatrix} \begin{bmatrix} P_M \\ P_S \end{bmatrix} \tag{23}$$

The substitution of Eq. (23) into Eq. (16) leads to

$$\begin{bmatrix} I + R_{SC}^T M_S R_{SC} & R_{SC}^T M_S T_S \\ T_S^T M_S R_{SC} & I \end{bmatrix} \begin{bmatrix} \ddot{P}_M \\ \ddot{P}_S \end{bmatrix} + \begin{bmatrix} \Lambda_M & 0 \\ 0 & \Lambda_S \end{bmatrix} \begin{bmatrix} P_M \\ P_S \end{bmatrix}$$

$$= \begin{bmatrix} T_M^T B_M + T_C^T B_C + R_{SC}^T B_S \\ T_S^T \end{bmatrix} U \tag{24a}$$

$$Y = [C_{PM} T_M + C_{PC} T_C + C_{PS} R_{SC}, C_{PS} T_S] \begin{bmatrix} P_M \\ P_S \end{bmatrix}$$

$$+ [C_{VM} T_M + C_{VC} T_C + C_{VS} R_{SC}, C_{VS} T_S] \begin{bmatrix} \dot{P}_M \\ \dot{P}_S \end{bmatrix} \tag{24b}$$

The mass matrix for the amplitudes P_M of the approximate normal modes of vibration for the whole spacecraft is not diagonal because of the static deformation R_{SC} in the subbody; the amplitudes P_M are not orthogonal to each other. In order to make the amplitudes P_M orthogonal, we introduce the following transformation:

$$P_M = \hat{T}_M \hat{P}_M \tag{25}$$

where

$$\Lambda_M \hat{T}_M = (1 + R_{SC}^T M_S R_{SC}) \hat{T}_M \hat{\Lambda}_M$$

$$T_M^T (I + R_{SC}^T M_S R_{SC}) T_M = I$$

On the other hand, when the mass of the subbody is small compared with that of the whole spacecraft, the amplitudes P_M also become approximately orthogonal to each other by the following transformation:

$$(\hat{T}_M)_{ij} = \frac{\lambda_{Mi}^2}{(\lambda_{Mj}^2 - \lambda_{Mi}^2)(R_{SC}^T M_S R_{SC})_{ij}} \tag{25′}$$

$$(\hat{T}_M)_{ii} = 1$$

The substitution of Eq. (25) into Eq. (24) leads to

$$\begin{bmatrix} I & \hat{M}_{MS} \\ (\text{sym}) & I \end{bmatrix} \begin{bmatrix} \ddot{\hat{P}}_M \\ \ddot{P}_S \end{bmatrix} + \begin{bmatrix} \hat{\Lambda}_M & 0 \\ 0 & \Lambda_s \end{bmatrix} \begin{bmatrix} \hat{P}_M \\ P_S \end{bmatrix} = \begin{bmatrix} \hat{B}_M \\ \hat{B}_S \end{bmatrix} U \tag{26a}$$

$$Y = [\hat{C}_{PM}\ \hat{C}_{PS}] \begin{bmatrix} \hat{P}_M \\ P_S \end{bmatrix} + [\hat{C}_{VM}\ \hat{C}_{VP}] \begin{bmatrix} \dot{\hat{P}}_M \\ \dot{P}_S \end{bmatrix} \tag{26b}$$

where

$$\hat{M}_{MS} = \hat{T}_M^T R_{SC}^T M_S R_{SC}$$

$$\hat{B}_M = \hat{T}_M^T (T_M^T B_M + T_C^T B_C + R_{SC}^T B_S)$$

$$\hat{B}_S = \hat{T}_S^T$$

$$\hat{C}_{PM} = (C_{PM} T_M + C_{PC} T_C + C_{PS} R_{SC}) \hat{T}_M$$

$$\hat{C}_{PS} = C_{PS} T_S$$

$$\hat{C}_{VM} = (C_{VM} T_M + C_{VC} T_C + C_{VS} R_{SC}) \hat{T}_M$$

$$\hat{C}_{VS} = C_{VS} T_S$$

Equations (26) are expressed in terms of the orthogonal coordinates $\hat{P}_M$ and P_S, where coordinates $\hat{P}_M$ and P_S describe the displacements of the whole spacecraft and subbody, respectively. The coordinates $\hat{P}_M$ and P_S interact with each other through the mass matrix $\hat{M}_{MS}$. A reduced-order model is derived by deleting certain components of the normal modes of vibration for the whole spacecraft and subbody. The effect of structural damping is taken into account in the reduced-order model as modal damping models of the normal modes of vibration for the whole spacecraft and subbody. For the general case including any number of subbodies, the subbody represents all other subbodies of the spacecraft coupled to the main body. This reduced-order model is well suited to the design of a control system of a spacecraft composed of a main body and subbody with mission equipment. The control system of this class of spacecraft usually consists of the global and local controllers; the former controls the attitude of the whole spacecraft and the latter the attitude of a local part, i.e., the attitude of mission equipment. The coordinates P_M are used for the design of a global controller and the coordinates P_S for the design of a local controller. At the design phase, the structures of the subbodies are frequently changed. When the structures of the subbodies are changed, the reduced-order model proposed requires recalculation of the normal modes of vibration and the static deformations for the subbodies, Eqs. (20) and (18), and the normal modes of vibration for the whole spacecraft, Eq. (25). In spite of the case of the reduced-order model in terms of the normal modes of vibration for the whole spacecraft, the calculation of the normal modes of vibration for the whole spacecraft, Eq. (25), does not require an eigenvalue problem with large dimensions for a wide frequency region. Moreover, if the mass of the subbodies is small compared with that of the whole spacecraft, an analytical solution Eq. (25′) gives a good approximation to Eq. (25).

IV. Numerical Examples

The reduced-order models proposed are applied to a simple spacecraft model. The spacecraft model is shown in Fig. 3; the spacecraft consists of a plate and tower, and the systems are frame structures and the components are beams. The first reduced-order model is examined. The rotational displacements θ_{ya}, θ_{yb} about the Y axis at points a, b, in Fig. 3 are chosen to be the outer coordinates; the sensors and actuators are assumed to be located at points a and b. Then, the reduced-order model expresses the transfer functions between these points accurately in a low-frequency region. The modulus of the transfer function between θ_{yb} and θ_{yb} based on the reduced-order model proposed is shown in Fig. 4, where 10 normal modes of vibration are retained by the use of the criterion in Eq. (14′). For comparison, the result obtained from the reduced-order model is described in the normal modes of vibration is shown in Fig. 5, where 12 normal modes

Table 1 Parameter values of a spacecraft model

Plate	Length	50 × 100 m
Tower	Height	30 m
Beam	Mass	16.7 kg/m
	Bending rigidity	1.93×10^6 Nm^2
	Torsional rigidity	1.59×10^8 N

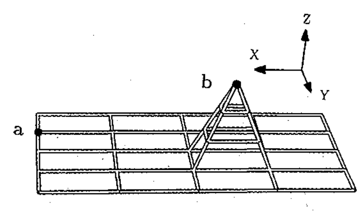

Fig. 3 Model of a simple spacecraft.

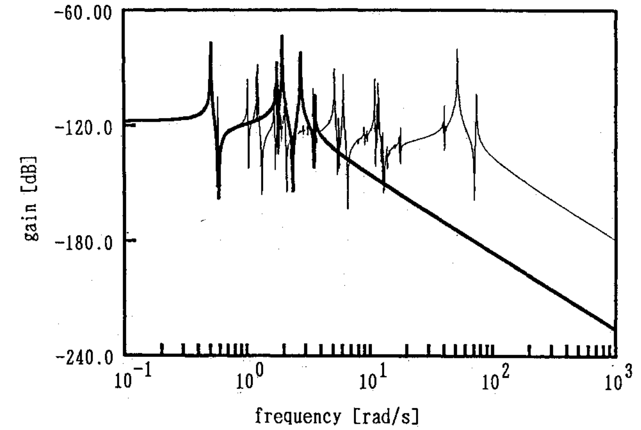

Fig. 4 Modulus of transfer function at θ_{yb}: Reduced-order model in terms of the modes of static deformation and the normal modes of vibration.

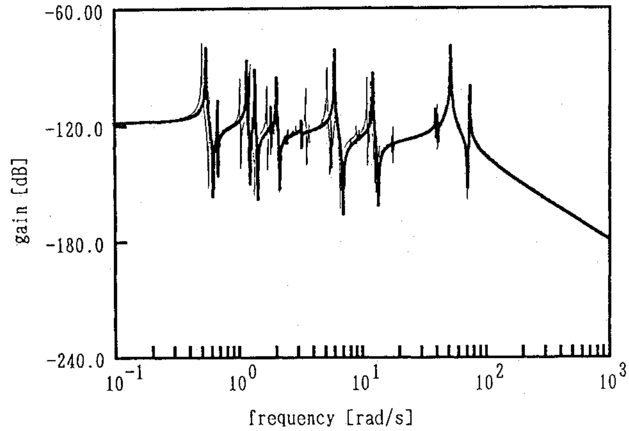

Fig. 6. Modulus of transfer function at θ_{yb}: Reduced-order model in terms of the global and local normal modes of vibration (nominal case).

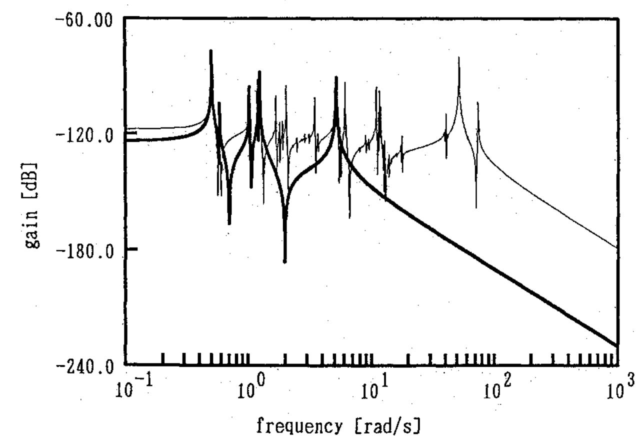

Fig. 5 Modulus of transfer function at θ_{yb}: Reduced-order model in terms of the normal modes of vibration.

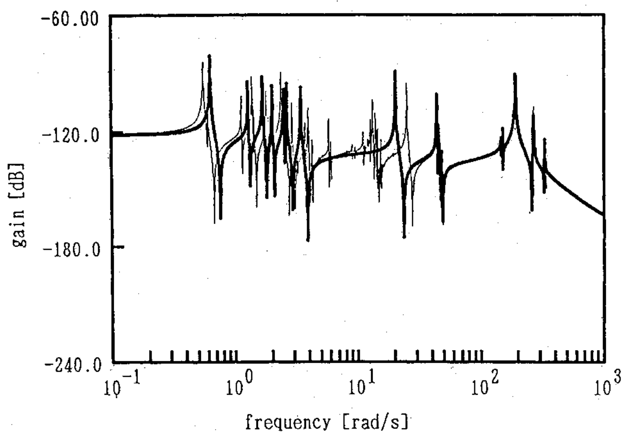

Fig. 7 Modulus of transfer function at θ_{yb}: Reduced-order model in terms of the global and local normal modes of vibration (mass density of the beams in the tower is decreased).

of vibration are retained by the use of modal cost analysis. In Figs. 4 and 5, the effects of the normal modes with zero frequency are omitted and the damping ratio for each normal mode is set at 0.001, and thin lines show the results of the exact model without truncation. The reduced-order model proposed shows a small discrepancy with the exact model in a low-frequency region; the model expresses the dynamic behavior of the system accurately in this region.

Next, the second reduced-order model is examined. The plate is considered the main body and the tower the subbody. The sensor and actuator are assumed to be located at the top of the tower; they are presumably used to control the attitude of the whole spacecraft and the attitude of the tower. Then, the reduced-order model must express the displacement of the whole spacecraft at a low-frequency region and the displacement of the tower accurately. The modulus of the transfer function at θ_{yb} based on the reduced-order model proposed is shown in Fig. 6, where 15 normal modes of vibration for the whole system and 10 normal modes of vibration for the subbody are retained by the use of modal cost analysis. In Figs. 6 and 7, the effects of the normal modes of vibration with zero frequency are omitted and the damping ratio for each normal mode is set at 0.001. Thin lines show the result of the exact model without truncation. In Fig. 6, the thin line shows that the displacement modes of the whole system with a large influence on the sensor and actuator exist near 1 rad/s, and the displacement modes of the tower exist above 5 rad/s. The thick line shows that the reduced-order model proposed expresses well the dynamic behavior of the global and local displacements of the system. Figure 7 shows the result when the value of the mass density of the beams used in the tower is decreased to 18% of the nominal one. The thin line shows that the displacement modes of the tower shift above 10 rad/s, and the thick line indicates that the reduced-order model reconstructed by only solving Eqs. (18), (20), and (25) also expresses the dynamic behaviors of the system well.

V. Conclusions

Two reduced-order models of a large flexible spacecraft have been proposed. These reduced-order models are based on a component mode synthesis. The first one is expressed in terms of the modes of static deformation and the normal modes of vibration for the spacecraft, which can express the dynamic behavior of the spacecraft accurately in a low-frequency region. The second one is expressed in terms of two sets of the normal modes of vibration, the approximate normal modes of the whole spacecraft, and the normal modes of the subbodies. The reduced-order model is suited to the design of a control system that consists of global and local controllers. The authors wish to express their thanks to Dr. Ohkami, Mr. Kida, and Mr. Yamaguchi of National Aerospace Laboratory of Japan for helpful discussions throughout the course of this work.

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